

# Finite Difference Time Domain (FDTD) Simulations of Electromagnetic Wave Propagation Using a Spreadsheet

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We describe a simple and intuitive implementation of the method of finite difference time domain simulations for propagating electromagnetic waves using the simplest possible tools available in Microsoft Excel. The method overcomes the usual obstacles of familiarity with programming languages as it relies on little more than the cut and paste features that are standard in Excel. Avenues of exploration by students are proposed and sample graphs are included. The pedagogical effectiveness of the implementation was tested during an Independent Activities Period class, composed of 80% freshmen, at MIT, and yielded positive results.

## I. INTRODUCTION

Here we outline a simple demonstrative example of the method of finite difference time domain (FDTD) simulations of electromagnetic wave propagation appropriate for undergraduates in an introductory electricity and magnetism course or advanced high school science students using Microsoft Excel, a robust software package that offers advanced graphics, numeric, and animation capabilities requiring minimal computer experience.<sup>1,2,3</sup> Only two rows in the spreadsheet, one for initial electric field and another for initial magnetic field, need be specified in order to initiate the simulation, and graphs of the spatially and temporally evolving waveforms can be updated in real time as the simulation is carried out in Excel. We begin with a review of the basic tenets of the FDTD method, proceed into an outline for the implementation in Microsoft Excel, and conclude with some suggested exercises.

## II. METHODS

### A. FDTD basics

The starting point for an FDTD simulation are Maxwell's equations, which are repeated here for the case of one dimensional free space propagation in time ( $t$ ) and space ( $z$ ) with no sources or sinks for magnetic or electric fields  $\mathbf{B}$  or  $\mathbf{E}$  respectively for the corresponding material responses  $\mathbf{H}$  or  $\mathbf{D}$ ,

$$\frac{\partial E_x}{\partial t} = -\frac{1}{\epsilon_0} \frac{\partial H_y}{\partial z}, \quad (1)$$

$$\frac{\partial H_y}{\partial t} = -\frac{1}{\mu_0} \frac{\partial E_x}{\partial z}, \quad (2)$$

$$\nabla \cdot \mathbf{E} = 0, \quad (3)$$

$$\nabla \cdot \mathbf{B} = 0. \quad (4)$$

If we choose the x-direction for the polarization of the electric field and the z-direction for direction of propagation, then it follows that the magnetic field is y-polarized as indicated in the subscripts on the fields above. To discretize the equations of propagation, central difference approximations are taken for the derivatives of time and space. Temporal steps are indexed by the integer  $n$  and related to continuous time by the relation  $t = n\Delta t$ , and spatial steps are indexed by the integer  $k$  and related to continuous space by the relation  $z = k\Delta z$ . The temporal discretization method using central differences can then be designated for variable  $\mathbf{X}$  as

$$\frac{\partial X}{\partial t} \longrightarrow \frac{\Delta X}{\Delta t} \equiv \frac{X(n+1/2) - X(n-1/2)}{\Delta t}, \quad (5)$$

and spatial discretization as

$$\frac{\partial X}{\partial z} \longrightarrow \frac{\Delta X}{\Delta z} \equiv \frac{X(k+1/2) - X(k-1/2)}{\Delta z}. \quad (6)$$

In FDTD, we need only consider the two curl equations 1 and 2 above because the divergence conditions 3 and 4 can be satisfied implicitly by interleaving the electric and magnetic field components in space in what has come to be known as a Yee cell.<sup>4</sup> A consequence of the spatial interleaving is that the fields must also be interleaved in time, known as "leapfrog", since the temporal response of one field is proportional to the spatial variation of the other at the previous time step. Implementing eqs. (1) and (2) as indicated in eqs. (5) and (6) and interleaving spatially and temporally yields the difference equation version of Maxwell's equations:

$$\frac{E_x^{n+\frac{1}{2}}(k) - E_x^{n-\frac{1}{2}}(k)}{\Delta t} = -\frac{H_y^n(k+\frac{1}{2}) - H_y^n(k-\frac{1}{2})}{\epsilon_0 \Delta z}, \quad (7)$$

$$\frac{H_y^{n+1}(k+\frac{1}{2}) - H_y^n(k+\frac{1}{2})}{\Delta t} = -\frac{E_x^{n+\frac{1}{2}}(k+1) - E_x^{n+\frac{1}{2}}(k)}{\mu_0 \Delta z}. \quad (8)$$

At a given position in space, the field at each current time step can be calculated from the previous values of the fields. Solving for the latest field at the latest time step in eqs. (7) and (8) yields:

$$E_x^{n-\frac{1}{2}}(k) - \frac{\Delta t}{\epsilon_0 \Delta z} \{H_y^n(k + \frac{1}{2}) - H_y^n(k - \frac{1}{2})\}, \quad (9)$$

$$H_y^{n+1}(k + \frac{1}{2}) = H_y^n(k + \frac{1}{2}) - \frac{\Delta t}{\mu_0 \Delta z} \{E_x^{n+\frac{1}{2}}(k+1) - E_x^{n+\frac{1}{2}}(k)\}. \quad (10)$$

In this fashion, known as Euler forward, the solution of Maxwell's equations proceeds much the same way that we envision electromagnetic wave propagation—an electric field induces a magnetic field, which induces an electric field, ad infinitum. The time step and grid size parameters are chosen based on the propagating wave frequency and wavelength. For stability, the general rule is that at least ten grid points sum to less than the smallest wavelength  $\lambda_{min}$  considered, and the Courant condition then determines the time step.<sup>5</sup>

$$\Delta z \leq \frac{\lambda_{min}}{10}, \quad (11)$$

$$\Delta t \leq \frac{\Delta z}{2c_0}. \quad (12)$$

where  $c_0$  is the speed of light in vacuum.

The computer algorithm implementation of this procedure, eqs. (9) and (10), requires two one-dimensional arrays in the spatial coordinate, one for  $E_x$  and one for  $H_y$ , to be allocated in memory. After imposing an initial condition, in the form of an initial electric or magnetic field, each time step is calculated by the following prescription:

$$\begin{aligned} t &= t_0 + n \frac{\Delta t}{2} \\ \tilde{E}_x[k] &= \tilde{E}_x[k] + \frac{1}{2} \{H_y[k-1] - H_y[k]\} \\ t &= t_0 + n \frac{\Delta t}{2} \\ H_y[k] &= H_y[k] + \frac{1}{2} \{\tilde{E}_x[k] - \tilde{E}_x[k+1]\} \end{aligned} \quad (13)$$

where we have adopted the normalized fields ( $\tilde{E} = \sqrt{\epsilon_0 \mu_0} E$ ) to simplify the code, and included the stability conditions, eqs. (11) and (12), within the constant preceding the curl.<sup>6</sup> The algorithm (eqn. (13)) is iterated for the desired number of time steps. Note that iteration of (13) results in an implicit time formulation, i.e. time is not made explicit in the equations for the field and only appears in the algorithm above for bookkeeping purposes.

## B. FDTD spreadsheet algorithm

The FDTD methodology for one dimension can be implemented in spreadsheet format, here in Microsoft Excel, using simple cell formulas and cut-and-paste features. This has an advantage over other pedagogical approaches<sup>7</sup> in that no programming experience is required. The starting point is the algorithm in (13) where columns represent spatial steps and pairs of rows (one for E and one for H) represent time steps. The algorithm is illustrated graphically in the center section of figure 1. For any time step (any pair of rows of  $E$  and  $H$ ), the mapping of (13) is:  $E_x(\text{column})/H_y(\text{column})$ . Since the electric field value from one prior cell is needed to compute the curl,  $E_x$  begins at column one and ends at column  $k+1$ . Similarly, the magnetic field begins at column zero but ends at column  $k$ , since its value at one subsequent cell is needed when computing the curl. The first two rows ( $t=0$ ) are for the initial conditions. The algorithm computes the remaining cells. The student need only type in the formulas for  $E$  and  $H$  for the second time step and the first two columns; cut and paste may be used to complete the remaining spatial columns and temporal rows.

The procedure for implementing the algorithm is as follows:

1. As a visual aid, enter the spatial coordinate (1, 2, 3, etc) in the topmost row of the spreadsheet starting at column  $B$ . After typing in the first two spatial coordinates, select cells  $B$  and  $C$  and drag the "fill" handle to encompass as large a problem space as needed.
2. Type 'Ex0' in cell  $A2$  and 'Hy0' in cell  $A3$ , where the number following the field component refers to the time step. Drag the "fill" handle down the column, as in 1, to include the desired number of time steps.
3. Highlight the leftmost and rightmost columns of the problem space.
4. Fill in all of row 2 and 3 (time step 0) to the end of the problem space with zeros and highlight to indicate that this is the initial condition for the fields.
5. In the leftmost column and first time step for Ex, type '=B2'. Do the same for the rightmost column. This imposes a perfect electric conductor radiation boundary condition on the problem space, which means that impinging waves will reflect from the problem space boundaries as they would from a mirror.
6. In the second spatial position of Ex for time step one (cell  $C4$ ), type the formula '=C2+0.5\*(B3-C3)' and press enter.

7. Select the cell from the previous step and drag the "fill" handle to the last un-highlighted spatial column in the row.
8. In the first spatial position of  $H_y$  for time step one (cell C5) type the formula '=B2+0.5\*(B4-C4)' and press enter. These last two steps define the computing steps of the algorithm.
9. Select the cell from the previous step, and drag the "fill" handle to the last un-highlighted spatial column in the row.
10. Finally, select all the columns for time step 1 ( $E_x$  and  $H_y$ ), and drag the "fill" handle through the last time step in the simulation. Test that it works by entering a 1 in the initial condition regions and see that all cells automatically update; note that the only valid regions for initialization are at the zero time step ( $E_x$  and/or  $H_y$ ) and between the first and last columns of the problem space (highlighted).

The FDTD Excel code will now update all fields in response to the initial conditions entered by the user. To graphically visualize the simulation output, a single row can be selected and graphed using the **insert > chart > (xy)scatter** menu item. This outputs the spatial field pattern at the moment in time designated by the row number. To graph temporal evolution at a single point in space, it is recommended to make a new field mesh with only one field component (either  $E_x$  or  $H_y$ ) on a separate sheet and graph from that; otherwise, both the E and H fields will appear in the graphs. This can be done using cut-and-paste. An example spatial and temporal graph of two counter-propagating pulses is illustrated in figure 3.

### III. APPLICATIONS OF THE ALGORITHM

#### A. Power flow

The direction of propagation of light is dictated by the phase between the electric and magnetic fields. For transverse electromagnetic (TEM) wave propagation the familiar right-hand rule applied by curling  $E$  into  $H$  indicates the direction of power flow. Using our spreadsheet code, this can be demonstrated by initializing both the electric and magnetic field as a sinusoidal waveform modulated by a Gaussian envelope. Introducing a phase shift of either 0 or  $\pi$  in the magnetic field allows the direction of propagation to be controlled. If the electric ( $E_x$ ) and magnetic ( $H_y$ ) field are both positive at the same points in space and time, then the pulse travels in the positive  $z$  direction; if the signs are opposite then propagation is in the opposite direction as illustrated in figure 2.

#### B. Guided wave optics

Some of the most interesting applications of electrodynamics occur when constraints are placed on the fields. The perfect electric conductor radiation boundary conditions form the walls of a resonator when the wavelength in the problem space (resonator width) is of similar size to the problem space itself. With PEC walls, no light wave with a wavelength longer than twice the resonator width will propagate. This is referred to as a cut-off wavelength. Also, since the electric field must be zero at the boundary and only nodes in a plane wave may satisfy this, only wavelengths that are half-integer multiples of the resonator width can propagate. Resonator modes can be investigated by setting the initial conditions to a plane wave with nodes at the boundaries is illustrated in figure 3.

#### C. Transmission and reflection

By introducing a relative permittivity  $\epsilon_r$  into equation (9), which changes the index of refraction and hence the wave propagation speed, the spreadsheet code can illustrate reflection and transmission at a material interface. In terms of the normalized fields, the constant  $1/2$  becomes  $1/2\epsilon_r$ . Small errors in the transmitted and reflected fields are to be expected, but these will decrease as the number of mesh points per wavelength increases. Figure 4 illustrates this process for an air ( $\epsilon_r = 1$ )/material ( $\epsilon_r = 2$ ) interface.

### IV. FURTHER EXERCISES AND SUGGESTED ACTIVITIES

The following exercises are recommended. The spreadsheet code for the examples is available online, but it is recommended that students augment the code on their own, because writing and tinkering with the code is the best way to understand how it works. Exercises are listed in order of increasing difficulty, or in a manner such that each exercise depends only on changes already administered.

#### A. Fundamental

1. Use trigonometric and exponential mathematical functions to generate the initial conditions and reproduce the results in figure 2.
2. Study the cavity modes of a one-dimensional resonator, figure 3, by introducing standing waves into the initial conditions. What happens when the wavelength becomes longer than the problem space size (the width of the resonator)?

3. Experiment with radiation boundary conditions. Change the radiation boundary condition from perfect electric conductor to perfect magnetic conductor ( $H_y$  is zero at the problem space boundaries). What is different about reflections from these two kinds of surfaces?
4. Implement a periodic boundary condition for unbounded propagation.
5. Reproduce the reflection and transmission simulation in figure 4, and then change the interface from air/material to material/air. What changes?

### B. Advanced

1. Experiment with spatially periodic variation in the relative permittivity. This should produce frequency-dependent filtering, and for some conditions should produce "photonic bandgap" materials with no propagation in certain frequency regions.
2. Introduce loss into the simulation by including non-zero conductivity in Maxwell's equations and implementing it into the FDTD code.
3. Implement an absorbing boundary condition.

## V. CONCLUSION

We have formulated a simple implementation of the FDTD method of propagating electromagnetic waves using basic features available in Microsoft Excel, and we have presented some illustrative examples. The implementation was tested on freshmen at MIT during its 2004

Independent Activities Period in a short course<sup>8</sup> that met for two hours on each of two consecutive days. This brief introduction was sufficient for students to gain substantial proficiency in elementary simulations like those illustrated above. The class enjoyed unusually high attendance, especially among students preparing for or recently involved in electromagnetism coursework. Instructor (D.W.W.) observation and student feedback after the course indicated considerable satisfaction with insights and proficiency gained. Several student responses pinpointed the graphical form of the spreadsheet based algorithm as the key merit of the approach. The benefit of keeping track of a small number of unit cells in order to see how the algorithm works is useful for beginning and intermediate students of computer simulation, numerical recipes, and electricity and magnetism. The automatic updating of Excel graphs makes this an effective learning tool for students interested in the basics of electromagnetic wave propagation, as the consequences of changes in initial conditions are illustrated graphically and instantly. Finally, the methodology learned in this rapid introduction may stimulate student interest in more advanced FDTD simulation techniques and their broad research applications.<sup>9,10</sup>

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<sup>10</sup> URL <http://nelson.mit.edu/>.

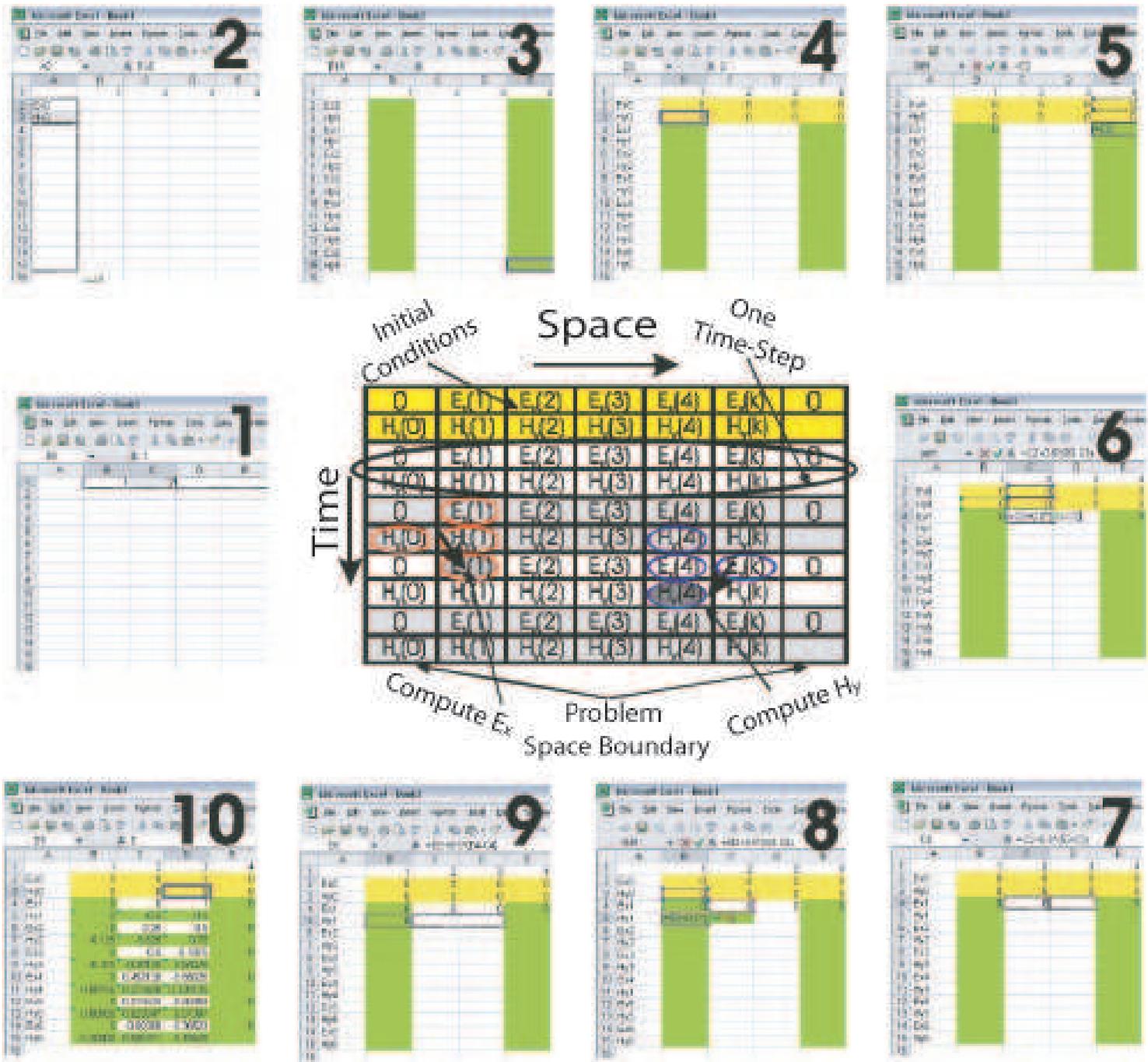


FIG. 1: Graphical illustration of the Spreadsheet implementation of FDTD. Center: Components of the spreadsheet fields and a graphical map of the field update dependencies. Exterior: Illustration of the implementation steps in the text.

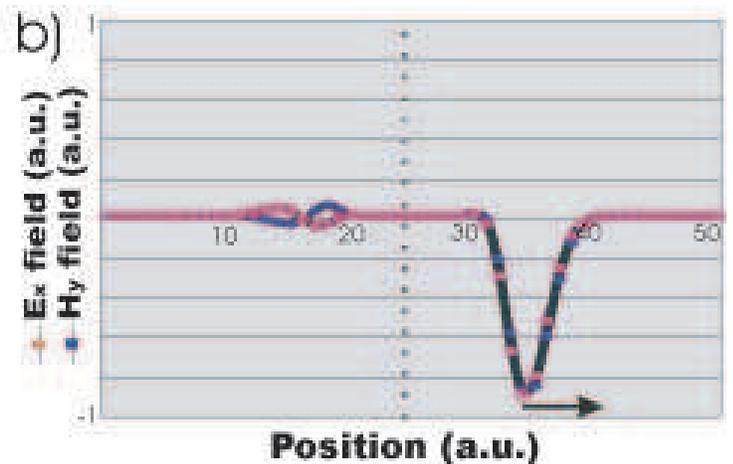
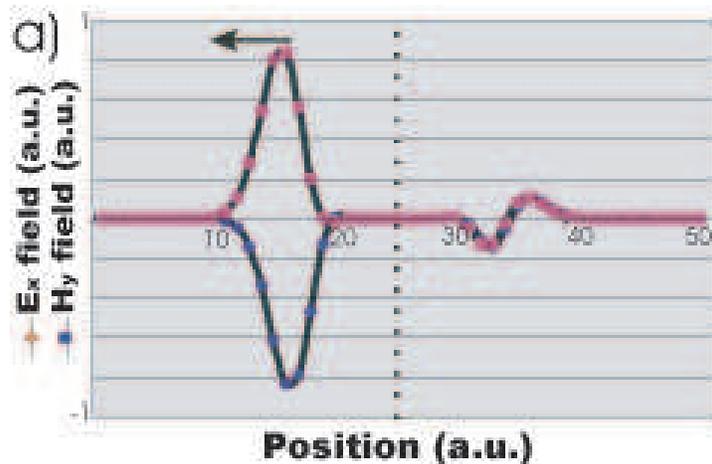


FIG. 2: Illustration of the relationship between the phases of E and H and the direction of propagation. a) Left-traveling pulse generated by introducing a  $\pi/2$  phase shift in the initial H field with respect to E. b) Right-traveling pulse generated by initializing E and H in phase with each other. Both graphs show the fields after the same (arbitrary) number of time steps after generation. The smaller pulses in a) and b) are remnants of imperfect initial conditions. The arrow indicates the direction of propagation and the dotted line down the center demarks the starting point of the pulse.

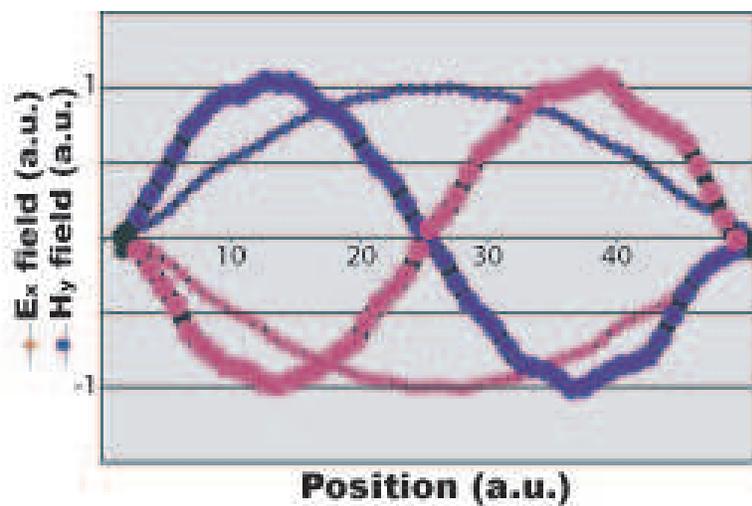


FIG. 3: Lowest energy symmetric and antisymmetric (bold) cavity mode for a 50 unit wide resonant cavity with perfect electric conductor walls. Figures displays the spatial distribution of the electric and magnetic fields in the cavity after 200 time steps.

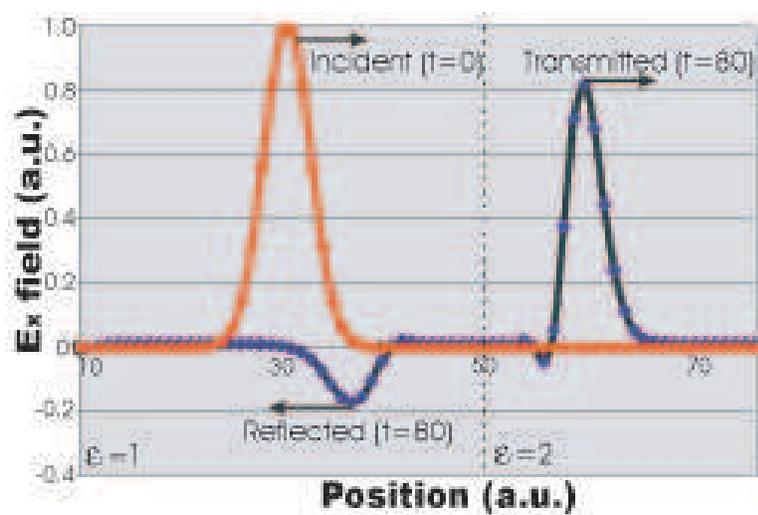


FIG. 4: Incident wave approaching an interface (dotted line), and transmitted and reflected waves moving away from it. Time is in units of integer time steps.