M E M O R A N D U M

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FROM: David M. Cook

SUBJECT: Suggested exercises for May Workshop

SUGGESTION #1: Particle Trajectories

The trajectory of a charged particle in a prescribed electromagnetic field satisfies the equation

$$m\frac{d^2\mathbf{r}}{dt^2} = q\mathbf{E} + q\mathbf{v}\times\mathbf{B}$$

with general initial conditions $\mathbf{r}(0) = \mathbf{r}_0$, $\mathbf{v}(0) = \mathbf{v}_0$. Let $\mathbf{E}$ and $\mathbf{B}$ be constant, and let $\mathbf{E}$ be perpendicular to $\mathbf{B}$.

(a) Cast the problem in a dimensionless form.

(b) Use your favorite computer algebra system (MAPLE, *Mathematica*, ...) to find the trajectory analytically, generate graphs of the trajectory under several initial conditions and relative magnitudes of $\mathbf{E}$ and $\mathbf{B}$, note the $\mathbf{E}\times\mathbf{B}$ drift and try to understand intuitively why it arises.

(c) Use IDL or Matlab or some other programming language to solve the problem numerically, generate graphs of the trajectory under several initial conditions and relative magnitudes of $\mathbf{E}$ and $\mathbf{B}$, note the $\mathbf{E}\times\mathbf{B}$ drift and try to understand intuitively why it arises.

SUGGESTION #2: The Electrostatic Potential of Charged Disks

The electrostatic potential $V(\mathbf{r})$ produced at an observation point $\mathbf{r}$ by a charge distribution composed of elements dq' located at $\mathbf{r}'$ is given by

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int\frac{dq'}{|\mathbf{r}-\mathbf{r}'|}$$

Consider a disk of radius a located in the xy plane with its center at the origin and carrying a uniform surface charge density σ. To set a notation, let

$$dq' = \sigma\rho'\,d\rho'\,d\phi' \quad ; \quad \mathbf{r} = x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}} + z\,\hat{\mathbf{k}} \quad ; \quad \mathbf{r}' = \rho'\cos\phi'\,\hat{\mathbf{i}} + \rho'\sin\phi'\,\hat{\mathbf{j}}$$

Throughout this exercise, use your favorite computer algebra system (MAPLE, *Mathematica*, ...).

(a) Starting with the given input information show that the electrostatic potential produced by the charged disk is given by

$$V(x,y,z) = \frac{1}{4\pi\epsilon_0}\int_0^a\int_{-\pi}^{\pi}\frac{\sigma\,\rho'\,d\rho'\,d\phi'}{[x^2+y^2+z^2+\rho'^2-2x\rho'\cos\phi'-2y\rho'\sin\phi']^{1/2}}$$

(b) Evaluate the integral in part (a) for an observation point on the z axis, i.e., show that the on-axis potential produced by the single charged disk is given by

$$V(0,0,z) = \frac{\sigma}{2\epsilon_0}\left(\sqrt{z^2+a^2} - |z|\right)$$

(c) Now, suppose that you have two identical disks, one positively charged and the other negatively charged. Each has its center on the z axis; the positively charged disk lies in the plane $z = c$ and the negatively charged disk lies in the plane $z = -c$. Show that the on-axis electrostatic potential produced by this pair is given by

$$V_{\text{pair}}(0,0,z) = \frac{\sigma}{2\epsilon_0}\left(\sqrt{(z-c)^2+a^2} - |z-c| - \sqrt{(z+c)^2+a^2} + |z+c|\right)$$

(d) Generate graphs of $V_{\text{pair}}(0,0,z)/(\sigma a/2\epsilon_0)$ versus z/a for various values of c/a and write a few paragraphs describing the results.

SUGGESTION #3: The Off-Axis Potential of a Charged Ring

The electrostatic potential $V(\mathbf{r})$ produced at an observation point $\mathbf{r}$ by a charge distribution composed of elements dq' located at $\mathbf{r}'$ is given by

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{dq'}{|\mathbf{r}-\mathbf{r}'|}$$

Consider a ring of radius a located in the xy plane with its center at the origin and carrying a uniform line charge density λ. To set a notation, let

$$dq' = \lambda\,\rho'\,d\phi' \quad ; \quad \mathbf{r} = x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}} + z\,\hat{\mathbf{k}} \quad ; \quad \mathbf{r}' = a\cos\phi'\,\hat{\mathbf{i}} + a\sin\phi'\,\hat{\mathbf{j}}$$

(a) Starting with the given input information, show that the electrostatic potential produced by the charged ring is given by

$$V(x,y,z) = \frac{\lambda a}{4\pi\epsilon_0}\int_{-\pi}^{\pi} \frac{d\phi'}{[x^2+y^2+z^2+a^2-2ax\cos\phi'-2ay\sin\phi']^{1/2}}$$

(b) Limit the observation point to a point in the xz plane and show that the integral in part (a) reduces to

$$V(x,0,z) = \frac{\lambda a}{2\pi\epsilon_0}\int_{0}^{\pi} \frac{d\phi'}{[x^2+z^2+a^2-2ax\cos\phi']^{1/2}}$$

(c) Use numerical integration to evaluate $V(x,z)/(\lambda/2\pi\epsilon_0)$ as a function of x/a for several different values of z/a ranging from $z/a = 0$ to $z/a = 2$, generate graphs of these functions, and write several sentences pointing out the significant features of these graphs.